

Reliability Analysis of Hysteretic Frame Subjected to Near-fault Ground Motions

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ABSTRACT: This study focuses on the dynamic reliability analysis of hysteretic frame structures subjected to near-fault impulsive ground motions. Firstly, a stochastic synthesis model of near-fault pulse-like ground motion is established. The long-period velocity pulse is fitted to the single Gabor wavelet with five random parameters by the least square method, and the low-frequency acceleration components are achieved by differencing the velocity pulse. Then, the high-frequency components are represented by a random function-based spectral representation method combining with a stochastic envelope function. Furthermore, the probability density evolution method (PDEM) is utilized to efficiently calculate the probability density functions of hysteretic dynamic responses of structures. To evaluate the first-passage dynamic reliability, an absorbing boundary condition is imposed in the generalized probability density evolution equation (GPDEE), so that the probability carried by the random event crossing the safety domain to be zero. Finally, the remarkable influences of fault distance and the occurrence instant of velocity pulse on dynamic reliability of hysteretic frames are scrutinized.

Keywords: hysteretic frame; dynamic reliability analysis; near-fault ground motions; stochastic synthesis model; probability density evolution method

1. INTRODUCTION

Near-fault ground motions are closely related with the rupturing mechanism of the fault, and generally present the effects of forward directivity and fling-step. Both the effects may cause the distinct long-period pulse in velocity time histories (Bray and Rodriguez-Marek 2004). Such velocity pulses possess larger amplitude and long duration and exhibit large uncertainty, which usually result in the nonlinear responses and damage for building structures. Therefore, the reliability assessment of building structures subject to near-fault ground motions is critical for seismic design of structures.

Many studies on the near-fault pulse model have been carried out. Commonly, a simple velocity waveform, such as the trigonometric functions (Mavroeidis and Papageorgiou 2003) and wavelet function (Baker 2007), is used to fit the long-period velocity pulse. Dickinson and

Gavin (2010) suggested a statistical analysis method for ground motion records under a seismic hazard level and a geographic region, and obtained the probability distribution of the parameters that describe the low- and high-frequency stochastic contents of ground motions. By taking the orientation of the strongest pulse into account, Yang and Zhou (2015) proposed a stochastic synthesis model of near-fault pulse-like ground motion, in which the velocity pulse was described by single Gabor wavelet. Dabaghi and Der Kiureghian (2017) suggested another model by combining M-P pulse model and filtered Gaussian random process, which can reflect the nonstationarity of ground motions. In above models, however, a lot of random variables are inevitably introduced to reproduce the near-fault impulsive ground motions. Such models have to make the Monte Carlo simulation (MCS) to assess the dynamic reliability of structures.

For dynamic reliability analysis, the probability density evolution method proposed by Li and Chen (2005) is an efficient and appropriate approach. This method involves the calculations of deterministic dynamic equation of structures and generalized probability density evolution equation (GPDEE). The dynamic reliability of structures can be obtained by imposing the absorbing boundary condition in GPDEE. However, a high-dimensional numerical integration is required, which restricts the number of random variables in stochastic input model.

This study aims to evaluate the dynamic reliability of building structures under near-fault ground motions. For this purpose, this work (i) establishes a stochastic model with nine random variables for near-fault impulsive ground motion suitable for PDEM; (ii) assesses the dynamic reliability of buildings; (iii) investigates the effects of velocity pulse parameters on dynamic reliability of structures.

2. STOCHASTIC SYNTHESIS MODEL OF NEAR-FAULT GROUND MOTION

2.1. Long-period Velocity Pulse

A stochastic pulse model with the Gabor wavelet established by Yang and Zhou (2015) to fit the strongest velocity pulse is utilized in this work, and expressed as follows

$$V_p(t; T_p, N_c, T_{pk}, \varphi, \sigma_{\ln PGV}) = PGV \cdot \exp \left[\sigma_{\ln PGV} - \frac{\pi^2}{4} \left(\frac{t - T_{pk}}{N_c T_p} \right)^2 \right] \times \cos \left(2\pi \frac{t - T_{pk}}{T_p} - \varphi \right) \quad (1)$$

where T_p , N_c , T_{pk} and φ represent the pulse period, number of circles in the pulse, the location and phase of the pulse, respectively; $\sigma_{\ln PGV}$ is the standard deviation of the regression residuals, which can be considered as random variable; the attenuation of PGV is fitted by using the regression formula presented by Bary and Rodriguez-Marekis (2004)

$$\ln(PGV) = c_1 + c_2 M_w + c_3 \ln(R^2 + c_4^2) + \sigma \quad (2)$$

where M_w is the moment magnitude; R is the fault distance; and c_1 , c_2 , c_3 and c_4 are the regression parameters; and σ represents the regression residual of Eq. (2), respectively.

2.2. High-frequency Components

The residual acceleration time series can be generated by extracting the domain pulse and differentiating the residual velocity history. In this study, a random variable based spectral representation method in Liu *et al.* (2016) is employed to simulate the residual stochastic nonstationary high-frequency components of near-fault ground motion, which is written as

$$a_{\text{res}}(t) = \sum_{k=0}^N \sqrt{S(t, \omega_k) \Delta \omega} \times [\cos(\omega_k t) X_k + \sin(\omega_k t) Y_k] \quad (3)$$

where $S(t, \omega_k)$ is the nonstationary power spectral density function of residual acceleration time history; $\Delta \omega = (\omega_u - \omega_l)/N$ denotes the frequency step size; $\omega_k = \omega_l + k(\omega_u - \omega_l)/N$ means the discrete frequency; X_k and Y_k are the orthogonal random variables, which can be defined as a random function with one elementary random variable γ as follows

$$X_k = \sqrt{2} \cos(k\gamma + \frac{\pi}{4})$$

$$Y_k = \sqrt{2} \sin(k\gamma + \frac{\pi}{4}) \quad (4)$$

in which the elementary random variable γ is uniformly distributed within $[-\pi, \pi]$. In Eq. (3), a modified K-T (Kanai-Tajimi) spectrum with high-pass filter modulated by a random variable based envelope function in Yang and Zhou (2015) is used to express the nonstationary spectral function, namely

$$S(t, \omega) = |e(t)|^2 G(\omega) S_{K-T}(\omega) \quad (5)$$

where $e(t)$, $G(\omega)$ and $S_{K-T}(\omega)$ indicate the envelope function, Butterworth filter, K-T spectrum, respectively. To present the variability

of envelope function, the following stochastic envelope function with three random parameters in Yang and Zhou (2015) is also adopted

$$e(\alpha, \beta, \tau; t) = \begin{cases} 0 & t \leq t_0 \\ \left(\frac{t-t_0}{\tau} \right)^\alpha & t_0 \leq t \leq t_0 + \tau \\ e^{-\beta(t-t_0-\tau)} & t \geq t_0 + \tau \end{cases} \quad (6)$$

In Eq. (6) the parameter t_0 describes the initial instant of non-zero ground motion, and the envelope parameters τ , α and β are considered as random variables.

As a result, the stochastic velocity time history $V_s(t)$ of ground motion scaled by the peak of residual velocity history V_{res} , and the high-frequency acceleration $a_s(t)$ of the near-fault ground motion can then be obtained by differentiating the scaled velocity time series. Finally, the acceleration time series with the strongest pulse can be generated by the superposition of the high-frequency acceleration and the low-frequency counterpart $a_p(t)$ achieved from the velocity pulse function shown in Eq. (1).

3. DYNAMIC RELIABILITY ANALYSIS OF HYSTERETIC FRAME USING PDEM

3.1. Generalized Probability Density Evolution Equations

This study focuses on hysteretic frame structures, and modified Bouc-Wen hysteretic model is considered. The differential equation of motion for the structure subjected to stochastic ground motions can be described as follows

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \alpha\mathbf{K}\mathbf{X} + (1-\alpha)\mathbf{K}\mathbf{Z} = -\mathbf{M}\mathbf{I}\ddot{u}_g(\boldsymbol{\theta}, t) \quad (7)$$

with the deterministic initial condition $\mathbf{X}(t)|_{t=0} = \mathbf{X}_0$, $\dot{\mathbf{X}}(t)|_{t=0} = \dot{\mathbf{X}}_0$. The deterministic initial condition \mathbf{X}_0 and $\dot{\mathbf{X}}_0$ are considered in this study. In Eq. (7), $\ddot{\mathbf{X}}$, $\dot{\mathbf{X}}$ and \mathbf{X} denote the acceleration, velocity and displacement vector, respectively; $\mathbf{M} = \text{diag}(m_1, m_2, \dots, m_{12})$ is the mass matrix; \mathbf{K} is initial stiffness matrix; Rayleigh damping, i.e., $\mathbf{C} = a\mathbf{M} + b\mathbf{K}$ is adopted in this study;

α is the ratio of the final tangent stiffness to initial stiffness; \mathbf{I} is the $n \times 1$ unit column vector; $\ddot{u}_g(\boldsymbol{\theta}, t)$ indicates the acceleration history of near-fault impulsive ground motion described in Section 2; $\boldsymbol{\theta} = (T_p, N_c, T_{pk}, \varphi, \sigma_{\ln \text{PGV}}, \alpha, \beta, \tau, \gamma)$ denotes the random parameters vector of near-fault ground motion; $\mathbf{Z} = (Z_1, Z_2, \dots, Z_{12})^T$ denotes the hysteretic displacement, where Z_i is expressed as the following first-order nonlinear differential equation with respect to the relative displacement between floors $u_i = x_i - x_{i-1}$

$$\dot{Z}_i = h(Z_i) \frac{A\dot{u}_i - \nu \left(\beta |\dot{X}_i| \cdot |\dot{Z}_i|^{N-1} Z_i + \gamma \dot{X}_i |\dot{Z}_i|^N \right)}{\eta} \quad (8)$$

where $\alpha, \beta, \gamma, A, N, d_v, d_\eta, \zeta_s, \psi, d_\psi, \lambda, p, q$ are parameters of Bouc-Wen model.

Based on the principle of probability conservation of random event description, Li and Chen (2012) derived the uncoupled generalized probability density evolution equation, and the one-dimensional GPDEE is expressed as follows

$$\frac{\partial p_{\mathbf{x}\boldsymbol{\theta}}(\mathbf{x}, \boldsymbol{\theta}, t)}{\partial t} + \dot{X}(\boldsymbol{\theta}, t) \frac{\partial p_{\mathbf{x}\boldsymbol{\theta}}(\mathbf{x}, \boldsymbol{\theta}, t)}{\partial x} = 0 \quad (9)$$

with boundary condition: $p_{\mathbf{x}\boldsymbol{\theta}}(\mathbf{x}, \boldsymbol{\theta}, t)|_{x \rightarrow \pm\infty} = 0$ and initial condition: $p_{\mathbf{x}\boldsymbol{\theta}}(\mathbf{x}, \boldsymbol{\theta}, t)|_{t=0} = \delta(x - x_0) p_{\boldsymbol{\theta}}(\boldsymbol{\theta})$, where $\delta(\cdot)$ is the Dirac delta function; x_0 denotes the initial value of the interested physical quantity $X(t)$, which is one component of the \mathbf{X}_0 ; $p_{\boldsymbol{\theta}}(\boldsymbol{\theta})$ is the joint probability density function of random parameters $\boldsymbol{\theta}$.

Subsequently, the joint probability density function (PDF) of $\mathbf{X}(t)$ can be achieved by

$$p_{\mathbf{X}}(\mathbf{x}, t) = \int_{\Omega_{\boldsymbol{\theta}}} p_{\mathbf{x}\boldsymbol{\theta}}(\mathbf{x}, \boldsymbol{\theta}, t) d\boldsymbol{\theta} \quad (10)$$

where $\Omega_{\boldsymbol{\theta}}$ is the distribution domain of $\boldsymbol{\theta}$. For a general structure, the numerical procedure is required to be implemented.

3.2. First-passage Dynamic Reliability Analysis
According to the first passage failure criterion, the dynamic reliability of structures is defined as

$$P_s(t) = \Pr \{ X(\tau) \in \Omega_s, 0 < \tau \leq t \} \quad (11)$$

where Ω_s denotes the safe domain.

By using PDEM, the dynamic reliability in Eq. (11) can be reformulated by imposing the following absorbing boundary condition in PDEE (Li and Chen 2005)

$$p_{\mathbf{x}\theta}(\mathbf{x}, \theta, t) = 0, \quad \mathbf{x} \notin \Omega_s \quad (12)$$

This condition means the probability information will never return to the safety domain once it exceeds the safety threshold. Combining Eqs. (9) and (12), the joint probability density function $\hat{p}_{\mathbf{x}\theta}(\mathbf{x}, \theta, t)$ can be calculated, the remaining probability density of responses then be achieved by

$$\hat{p}_{\mathbf{x}}(\mathbf{x}, t) = \int_{\Omega_\theta} \hat{p}_{\mathbf{x}\theta}(\mathbf{x}, \theta, t) d\theta \quad (13)$$

Consequently, the dynamic reliability of structure is formulated as

$$P_s(t) = \int_{\Omega_s} \hat{p}_{\mathbf{x}}(\mathbf{x}, t) d\mathbf{x} \quad (14)$$

4. NUMERICAL EXAMPLE

Herein, a 25-story hysteretic frame subjected to stochastic near-fault impulsive ground motion is considered to perform the dynamic reliability analysis. The stochastic model is established based on the pulse-like records in Chi-Chi earthquake ($M_w=7.6$), and the probability distribution of random parameters in the stochastic model are listed in Table 1.

Figure 1(a) illustrates the hysteretic curve of the structure subjected to near-fault impulsive ground motion. The PDF of the maximum inter-story drift in $t=60$ s is exhibited in Fig. 1(b). It is seen that the results obtained by PDEM agree well with those by MCS with 10000 sampling.

Figure 2(a) shows the dynamic reliability of the hysteretic frame under different threshold values (0.06, 0.08, 0.10, 0.12, 0.14 m), in which the results are also compared with those by MCS, and presents a good agreement of PDEM with MCS. Figure 2(b) examines the effect of occurrence instant T_{pk} of velocity pulse for near-fault ground motion with different fault distances on dynamic reliability of the structure. It is

observed that the reliability decreases remarkably from 1 to the minimum value in the time interval $[T_{pk} - 3\sigma_{T_{pk}}, T_{pk} + 3\sigma_{T_{pk}}]$, and the velocity pulse of near-fault ground motion affects significantly the dynamic reliability of structures, especially for the structures located in the area closer to the fault.

Table 1. The probability distribution of random parameters for impulsive ground motions.

Parameter type	Parameters	Distribution	Mean	Standard deviation
Pulse parameters	T_p	Normal	6.81	1.62
	N_c	Lognormal	0.01	0.29
	T_{pk}	Normal	22.32	6.41
	ϕ	Normal	3.06	1.71
	$\sigma_{\ln PGV}$	Normal	0.00	0.25
Envelope parameters	τ	Normal	20.43	6.68
	α	Lognormal	0.63	0.41
	β	Lognormal	-2.56	0.39
High-frequency parameter	γ	Uniform	0.00	$\pi^2/3$

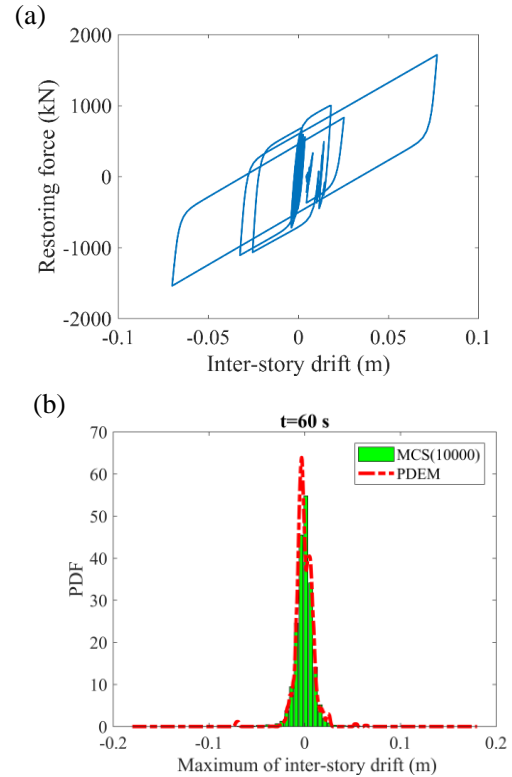


Figure 1. (a) Hysteretic curve under near-fault impulsive ground motion; (b) PDF of the maximum of inter-story drift at $t=60$ s

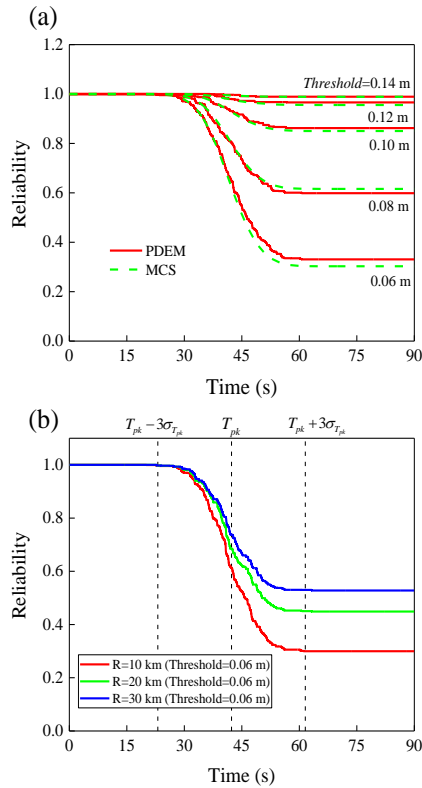


Figure 2. Dynamic reliability in different (a) threshold and (b) fault distance R

5. CONCLUSIONS

This study presents an efficient framework to perform the dynamic reliability analysis of hysteretic structure subjected to near-fault impulsive ground motions. Firstly, a stochastic synthesis model with 9 random parameters is established based on the actual records. Then, the probability density evaluation method with absorbing boundary condition is employed for dynamic reliability analysis. As an example, the dynamic reliability of the 25-story hysteretic frame is performed. The results show that the velocity pulse has a critical effect on failure of structures in the area closer to the rupturing fault; the variability of velocity pulse parameters plays an important role and cannot be neglected in structural dynamic reliability analysis; the proposed approach of dynamic reliability assessment for structure in near-fault area is efficient in contrast to MCS.

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